

Mass characteristics of active and sterile neutrinos in a phenomenological (3+1+2)-model

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On the basis of the experimental data allowing existence of the anomalies going beyond the minimally extended Standard Model with three active neutrinos with different masses, we consider the generalized model with three active and three sterile neutrinos, one of which is relatively heavy ((3+1+2)-model). We study the basic characteristics that are used to describe the massive active and sterile neutrinos, the methods to determine the absolute scale of neutrino masses and the neutrino mass estimates based on the available experimental data. Taking into account the possible contributions of the sterile neutrinos, the dependences of the neutrino mass characteristics on the sterile neutrino mass are plotted. The results obtained can be used to interpret and to predict the results of various neutrino experiments.

PACS numbers: 14.60.Pq, 14.60.St, 12.10.Kt, 12.90.+b

Keywords: neutrino oscillations, mixing parameters, CP -invariance, neutrino masses, neutrino mass observables, sterile neutrinos.

I. INTRODUCTION

Among the problems of modern neutrino physics the basic ones are the question about Majorana or Dirac nature of neutrinos, the problem of the number of different types of neutrinos and the precise measurements of absolute values of the masses and mixing parameters of neutrinos. To address these issues, active experimental and theoretical studies of both the neutrino mass observables that define the absolute mass scale of neutrino and neutrino oscillation characteristics that characterize the mixing of neutrinos with different masses are currently performed. It is expected that a comprehensive solution of the problems of the nature of neutrinos, their number and values of the mixing parameters will be given in the Grand Unification Theory (GUT), which does not still exist now in the generally accepted version [1]. To identify directions for further development of the existing Standard Model (SM), various phenomenological models, which use precise experimental data obtained are proposed and studied. In the framework of these models, the approximate values of the masses and mixing parameters of neutrinos and the relationships between them can be obtained, which can play an important role in clarifying the ways to extend the SM and in further to build the GUT successfully. Of course, for realization of these aims the crucial role belongs to the data that will become available as a result of carrying out in the next few years the neutrino experiments such as PLANK, KATRIN, GERDA, CUORE, BOREXINO, Double CHOOZ, SuperNEMO, KamLand-Zen, EXO, etc.

Considering the results obtained recently in the neutrino physics, let us to pay a special attention to the

following two: the difference from zero (more than 5σ) of the reactor mixing angle θ_{13} [2] and the experimental evidence of the possible existence of new anomalies for neutrino and antineutrino fluxes in different processes [3]. The first result is of great importance for justification of the experimental search for the CP violation in the lepton sector [4] (because so far there are no experimental data to support the lepton CP violation [1]) and for definition of the explicit form of the neutrino mixing matrix and neutrino mass matrix [5]. The second result can be related to the sterile neutrinos, whose existence is not only beyond the SM, but also beyond the minimally extended SM with three active neutrinos with different masses (MESM). Sterile neutrinos, which do not interact with the known conventional particles can in principle be numerous. The most popular models for sterile neutrinos now are the phenomenological models with one or two sterile neutrinos. These are so-called (3 + 1)- and (3 + 2)-models [6, 7]. However, if we take into account a possible left-right symmetry of the weak interactions and identify sterile neutrinos with the right singlet neutrinos with respect to the $SU(2)_L$, then the number of sterile neutrinos should be three, and so we come to the less popular (3+3)-models. They are less popular for the two reasons: to explain the experimentally observed anomalies it is sufficient, as it is now considered, to introduce only two sterile neutrinos, and, in addition, recent data of the cosmological observations of the fluctuations of the cosmic microwave background barely allow us to add one additional type of neutrino, but to add extra one, not to speak about two new types of neutrino - this is a serious problem. On the other hand, as was noted above, the right-left symmetric models should operate with three sterile neutrinos [8–10]. Models with three sterile neutrinos have been considered in Ref. [11], while in Ref. [12] a model with a light sterile neutrino with a mass of the order of the mass of active neutrinos has been proposed. In this paper we consider the phenomenological (3 + 1 + 2)-model with one relatively heavy sterile neutrino and two

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light sterile neutrinos [8, 9]. In this model it is easy if necessary to reduce the number of sterile neutrinos to avoid the possible contradictions with the data of cosmological observations [13, 14]. However, there is still a possibility to keep the number of different sterile neutrinos equal to three, but for this it is necessary to reduce the link between the active and the sterile neutrinos and, for example, to decline the condition of thermal equilibrium between the sterile neutrinos and other relativistic particles in the early stages of the Universe [15–17].

In this paper, a $(3 + 1 + 2)$ -model is used for the possible solution of the two problems in the neutrino physics, namely, how to include the sterile neutrinos in a generalized model of weak interactions to explain the new detected anomalies in the neutrinos and antineutrinos fluxes in the various processes, and what a value takes the mass scale of the active and sterile neutrinos? As is known, the results of experiments involving neutrino oscillations can determine the values of differences of masses in square of these particles and not the mass values themselves that leads to the problem of neutrino mass hierarchy and the problem of their mass scale. These problems have become more complex due to the possible existence of sterile neutrinos with masses of the order of 1 eV. An extensive literature is devoted to consideration of these questions; note only some of the recent reviews [7, 18, 19].

The paper is organized as follows. In Section II, the main characteristics, which are used to describe the Dirac and Majorana massive neutrinos, as well as the available experimental data for these characteristics are given. The experimental evidences for the existence of anomalies that go beyond the Standard Model with three active neutrinos are discussed in Section III. The main statements of the phenomenological $(3 + 1 + 2)$ -model of neu-

trino are described in Section IV. In Section V, we present the phenomenological estimates of the masses of both active and sterile neutrinos. Particular attention is paid in Section VI to the methods of determination of the absolute scale of neutrino masses with the help of the mass observables and to the experimental limitations on possible values of the neutrino mass observables. In this section we also present the results of numerical calculations of the neutrino mass characteristics in the form of plots versus the possible values of the minimal neutrino mass, which take into account the contributions associated with the sterile neutrinos. In final Section VII we discuss the results of the paper, which can be used to interpret and to predict the results of various experiments on determination of the neutrino mass scale and on searching for the effects associated with sterile neutrinos.

II. OSCILLATION CHARACTERISTICS OF THE DIRAC AND MAJORANA MASSIVE NEUTRINOS

As is known, the oscillations of solar, atmospheric, reactor and accelerator neutrinos can be explained by mixing of the neutrino states. This means that the flavor neutrino states are a mixture of at least three massive neutrino states and vice versa. Neutrino mixing is described by the Pontecorvo–Maki–Nakagawa–Sakata matrix $U_{PMNS} \equiv U = VP$:

$$\psi_L^\alpha = U_i^\alpha \psi_L^i, \quad (1)$$

where ψ_L^α and ψ_L^i are the left chiral fields of flavor and massive neutrinos, respectively, with $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$. For the three active neutrinos, matrix V can be written in the standard parametrization as [1]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}, \quad (2)$$

where $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$, δ_{CP} is the phase associated with the Dirac CP violation in the lepton sector, $P = \text{diag}\{1, e^{i\alpha_{CP}}, e^{i\beta_{CP}}\}$, and α_{CP} and β_{CP} are the phases related to the Majorana CP violation.

In a general case, the unitary $n \times n$ -matrix is determined by n^2 real parameters, for which we may choose $n(n-1)/2$ angles and $n(n+1)/2$ phases. Taking into account the structure of the electroweak SM Lagrangian, which comprises the currents composed of the fields of quarks, charged leptons and neutrinos, in the case of the Dirac neutrino fields it is possible to eliminate the $2n-1$ phases. In the case when the neutrino fields are of the Majorana type, one can exclude only n phases associated with the Dirac charged leptons. In view of

this, $n \times n$ -matrix U depending on the nature of neutrinos is determined either by $n(n-1)/2$ angles and $(n-1)(n-2)/2$ phases if neutrinos are of the Dirac nature, or by $n(n-1)/2$ angles and $n(n-1)/2$ phases if neutrinos are the Majorana particles [20]. Thus, to set the mixing matrix U_{PMNS} in the case of three Dirac neutrinos it is necessary to determine the three mixing angles $\theta_{12}, \theta_{13}, \theta_{23}$ and one mixing phase (CP -phase δ_{CP}), while for three Majorana neutrinos one should determine the same three mixing angles ($\theta_{12}, \theta_{13}, \theta_{23}$) and three CP -phases δ_{CP}, α_{CP} and β_{CP} .

Because the CP violation in the lepton sector is not yet experimentally observed [1], in further we will consider the elements of the neutrino mixing matrix as real. Gen-

erally, when the number of different types of neutrinos is $3 + N$, the neutrino mass matrix M_ν can be determined with the help of the generalized mixing matrix \tilde{U} of rank $(3 + N) \times (3 + N)$ as follows

$$M_\nu = \tilde{U} M^d \tilde{U}^T, \quad (3)$$

where $M^d = \text{diag}\{m_1, m_2, m_3, \dots, m_{3+N}\}$ and m_i ($i = 1, 2, 3, \dots, 3 + N$) are the masses of the neutrinos. Thus, the matrix elements $M_{\nu,ij}$ depend on the values of the masses and mixing parameters of the neutrinos.

As is known, with using only the oscillation experiments with atmospheric, solar, reactor and accelerator neutrinos it is not possible to determine the absolute values of the neutrino masses, as well as their Dirac or Majorana nature. The obtained experimental results related to neutrino oscillations indicate a violation of the laws of conservation of lepton numbers L_e , L_μ and L_τ and, in addition, the existence of at least two non-zero and different neutrino masses. The latter is a consequence of the non-zero values of two oscillation parameters Δm_{12}^2 and Δm_{13}^2 (where $\Delta m_{ij}^2 = m_i^2 - m_j^2$). Three sets of experimental data are sensitive to the absolute scale of neutrino masses, namely, the experimental data on beta decay, the experimental data on neutrinoless double beta decay and the experimental data obtained as a result of cosmological observations of the structure of the early universe at large distances. With the help of appropriate data for each of these sets, respectively, one of three neutrino mass observables, namely, m_β , or $m_{\beta\beta}$, or m_Σ , which are defined below can be measured.

Here we display the values of the mixing angles and neutrino mass-squared differences obtained from the overall analysis of the latest high-precision measurements of the oscillation parameters, which determine three-flavor oscillations of light active neutrinos with the standard deviations at the level of 1σ [2]:

$$\sin^2 \theta_{12} = 0.307_{-0.016}^{+0.018}, \quad (4a)$$

$$\sin^2 \theta_{23} = \begin{cases} NH : 0.386_{-0.021}^{+0.024} \\ IH : 0.392_{-0.022}^{+0.039} \end{cases}, \quad (4b)$$

$$\sin^2 \theta_{13} = \begin{cases} NH : 0.0241_{-0.0025}^{+0.0025} \\ IH : 0.0244_{-0.0025}^{+0.0023} \end{cases}, \quad (4c)$$

$$\Delta m_{21}^2 / 10^{-5} \text{eV}^2 = 7.54_{-0.22}^{+0.26}, \quad (4d)$$

$$\Delta m_{31}^2 / 10^{-3} \text{eV}^2 = \begin{cases} NH : 2.47_{-0.10}^{+0.06} \\ IH : -2.46_{-0.11}^{+0.07} \end{cases}. \quad (4e)$$

Note that CP -phases α_{CP} , β_{CP} and δ_{CP} are not currently known, as well as the scale of neutrino masses. Since we know only the absolute value of Δm_{31}^2 , the absolute values of the neutrino masses can be ordered by two ways, namely: a) $m_1 < m_2 < m_3$ and b) $m_3 < m_1 < m_2$,

i.e., it can be realized, as they say, either the normal hierarchy (NH, in the case a) or the inverse hierarchy (IH, in the case b) of the neutrino mass spectrum.

III. ANOMALIES OF NEUTRINO FLUXES FROM DIFFERENT SOURCES

The results of a number of neutrino experiments at small distances (more exactly, at distances L , at which the numerical value of the parameter $L\Delta m^2/E$ is of the order of unity, with E the neutrino energy) indicate the possible existence of light sterile neutrinos with masses of the order of 1 eV [3]. Discovery of such neutrinos would be a fundamental contribution to the physics of weak interactions. Sterile neutrinos are involved to explain the experimental data, which can not be explained in terms of common three-flavor model of mixing of massive neutrinos. Firstly, it is the so-called LSND/MiniBooNE anomalies [21, 22]. Besides, it has recently been carried out refined calculations of the spectra of reactor antineutrinos [23], which result in higher calculated values of the fluxes of these particles. That is, the obtained experimental data indicate a deficiency of antineutrino fluxes under the measurements at distances from the source less than 100 m. Such distances from the source are called as small, and the experiments at such distances are called as SBL (shot baseline) experiments. Possible deficiency of reactor antineutrinos at low distances is called as the reactor anomaly [24]. Similar anomalies were observed in calibration measurements for experiments SAGE and GALLEX. Such anomalies are commonly called as calibration or gallium ones [25–27]. From this it follows that characteristic values of Δm^2 for sterile neutrinos are about 1eV^2 . Thus, currently available three types of anomalies for neutrino fluxes (LSND/MiniBooNE, reactor, gallic) can be interpreted as evidence on the level of about 3σ for the existence of sufficiently heavy sterile neutrinos. However, the additional verification of these anomalies are required.

Below we consider the phenomenological $(3 + 1 + 2)$ -model of the neutrino [8, 9, 28], which can be used for incorporation of sterile neutrinos in the formalism of the theory of weak interactions. This model contains three active light neutrinos and three sterile neutrinos, and one sterile neutrino is relatively heavy, while the other two are light sterile neutrinos. Assuming all the cosmological limitations on the total number of neutrinos [13, 14], the number of sterile neutrinos can be reduced in this model, if necessary. However, such a reduction should be stipulated by weighty reasons. Since the cosmological constraints on the total number of neutrinos obtained on the basis of observational data are model-dependent, more studies are needed concerning the development of the cosmological models that allow, at joint variation of other cosmological model parameters, the insertion of one to three new types of neutrinos (and neutrinos hardly interacting with the particles of the SM).

IV. THE BASIC POINTS OF THE PHENOMENOLOGICAL (3+1+2)-MODEL OF THE NEUTRINO

Consider the formalism of neutrino mixing for a generalized model of the weak interactions, in which there are three active neutrinos and three sterile neutrinos, i.e., the (3 + 3)-model. This model can be easily reduced to (3 + 2)- or even (3 + 1)-model, with reducing the number of sterile neutrinos. Different types of sterile neutrinos will be distinguished by indices x, y and z , and additional massive states will be distinguished by indexes $1', 2'$ and $3'$. The set of indices x, y and z will be denoted by common symbol a , and the set of indices $1', 2'$ and $3'$ will be denoted by symbol i' . Then the mixing matrix \tilde{U} of rank 6×6 can be represented in a block form with use of four 3×3 matrices S, T, V and W , so that

$$\begin{pmatrix} \nu_\alpha \\ \nu_a \end{pmatrix} = \tilde{U} \begin{pmatrix} \nu_i \\ \nu_{i'} \end{pmatrix} \equiv \begin{pmatrix} S & T \\ V & W \end{pmatrix} \begin{pmatrix} \nu_i \\ \nu_{i'} \end{pmatrix}. \quad (5)$$

Neutrino masses will be given by a set $\{m\} = \{m_i, m_{i'}\}$, in which $\{m_i\}$ will be arranged in the normal order as $\{m_1, m_2, m_3\}$, while $\{m_{i'}\}$ will be arranged in the reverse order as $\{m_{3'}, m_{2'}, m_{1'}\}$. In the current paper, the unitary matrix \tilde{U} will not be considered in the most general form, but we restrict ourselves only by special cases involving additional assumptions [28]. For example, on the basis of available data of laboratory and cosmological observations it can be assumed that the mixing between the active and the sterile neutrinos is small, and, moreover, as the basis of the massive sterile states we choose the states, for which the matrix W is the unit matrix. With keeping in the unitarity condition only the first-order terms, which are contained in small 3×3 matrices b and ΔU_{PMNS} , we can write the matrices S, T and V as

$$S = U_{PMNS} + \Delta U_{PMNS}, \quad (6a)$$

$$T = b, \quad V = -b^T U_{PMNS}. \quad (6b)$$

The contributions from the matrix elements of ΔU_{PMNS} in fact are already taken into consideration by using the experimental uncertainties of the matrix U_{PMNS} itself. Taking into account that in explaining the discovered anomalies in the neutrino fluxes the most attention should be paid to the adjustments of the neutrino flavor states ν_e and ν_μ , which are associated with the most massive sterile states, the matrix b can be chosen in the following manner: in the IH-case

$$b_{IH} = \begin{pmatrix} \gamma & \delta & \delta \\ \beta & \delta & \delta \\ \alpha & 0 & 0 \end{pmatrix}, \quad (7)$$

whereas in the NH-case

$$b_{NH} = \begin{pmatrix} 0 & 0 & \alpha \\ \delta & \delta & \beta \\ \delta & \delta & \gamma \end{pmatrix}, \quad (8)$$

where on the basis of the available experimental constraints [7, 18, 19, 29] the parameters α, β, γ and δ are small quantities lesser than 0.2 in absolute value. In addition, marking out among the mass of sterile neutrinos the greatest one we will consider that the masses of other sterile neutrinos are smaller by at least one order of magnitude. This choice of the masses is consistent with the estimates of the masses of sterile neutrinos given in the next section. Therefore, taking into account the accepted mass distribution of the sterile neutrinos, the (3 + 3)-model in this case can be called as the (3 + 1 + 2)-model of active and sterile neutrinos.

V. PHENOMENOLOGICAL ESTIMATES OF NEUTRINO MASSES

As is known, the problem of strongly differing masses of the fundamental fermions remains still unsolved. In the SM these masses arise from the Yukawa couplings between the fields of the fundamental fermions and the Higgs field. However, the values of neutrino masses are so small that probably the mechanism of their formation is mainly due to the Majorana nature of neutrinos. In this case, the main task is to determine the special mechanism of forming the masses of the Majorana neutrinos. In the absence of a satisfactory theory of this phenomenon the problem was considered at the phenomenological level by many authors [8, 9, 30–33]. We will use the results of Refs. [8, 28], in which the following estimates of the absolute values of the masses of the light active neutrinos m_i ($i = 1, 2, 3$) were obtained. For the NH-case, they are [in eV]

$$m_1 \approx 0.0015, \quad m_2 \approx 0.0088, \quad m_3 \approx 0.0497, \quad (9a)$$

and for the IH-case they are

$$m_1 \approx 0.0496, \quad m_2 \approx 0.0504, \quad m_3 \approx 0.0020. \quad (9b)$$

Mass estimates for the right-handed neutrinos M_i for these two cases result in the following values, respectively:

$$M_1 \approx \Lambda_1, \quad M_2 \approx 0.002, \quad M_3 \approx 0.002, \quad (10a)$$

$$M_1 \approx 0.002, \quad M_2 \approx 0.002, \quad M_3 \approx \Lambda_3, \quad (10b)$$

where $\Lambda_{1,3}$ are the free parameters of the order of 1 eV. As was noted above, it is possible to compare the right neutrinos ν_{Ri} ($i = 1, 2, 3$) to the sterile neutrinos $\nu_{i'}$ ($i' = 1', 2', 3'$), i.e., to equate masses M_i to masses $m_{i'}$. It results in separation of the three sterile neutrinos with respect to masses into two light and one relatively heavy neutrino. Light right sterile neutrinos can be combined with light left neutrinos and form quasi-Dirac neutrinos. The case with three light active neutrinos, one heavier sterile neutrino and two light sterile neutrinos was called

in Refs. [8, 9] as the $(3+2+1)$ -model. This model can be reduced, if necessary, to $(3+1+1)$ - and even $(3+1)$ -model, eliminating the light right-handed neutrinos, which are singlets of the gauge group $SU(2)_L \times U(1)_Y$ of the weak and electromagnetic interactions.

The given above values of neutrino masses can be used, for example, for estimates of the values of the anomalous magnetic moments of neutrinos [34]. As a part of the MESM with non-zero neutrino masses, the neutrino magnetic moments are due to the radiative corrections. In general, the magnitudes of the magnetic moments, which are proportional to the masses of neutrinos form a matrix, the size of which depends on the number of neutrino flavours, and for the Majorana neutrinos the diagonal magnetic moments equal to zero. The diagonal Dirac moments in the one-loop approximation are as follows [35, 36]

$$\mu_i = (3G_F m_e m_i / 4\pi^2 \sqrt{2}) \mu_B \quad (i = 1, 2, 3), \quad (11)$$

where μ_B is Bohr magneton, m_e is mass of the electron, G_F is Fermi constant of the weak interactions, i.e., $\mu_i / \mu_B \approx 3.203 \times 10^{-19} \times m_i [\text{eV}]$.

Using estimates of the neutrino masses given in Eqs. (9)–(10), when the right-handed neutrinos are identified with light sterile neutrinos, we find that the massive Dirac neutrinos must have magnetic moments of the order of $10^{-20} \mu_B$. In the MESM, the off-diagonal magnetic moments of neutrino are suppressed by several orders of magnitude as compared with the diagonal magnetic moments. However, there are some models that go beyond the MESM, in which the off-diagonal Majorana neutrino magnetic moments can be much larger. The most optimistic in this respect are extended models with restored left-right gauge symmetry. For example, in such models for the off-diagonal neutrino magnetic moments $\mu_{e\tau}$ and $\mu_{\mu\tau}$ the estimates are obtained on the level of about $(10^{-13} \div 10^{-14}) \mu_B$, and these values depend on the mixing angle between the left and right gauge W -bosons [37]. The use of such models is most justified in interpreting the sterile neutrinos as a right-handed neutrinos, which is adopted in this paper.

The above estimates of relatively large magnetic moments of Majorana neutrinos in the framework of the right-left symmetric models, if they are realized, will affect, for example, the dynamics of flavour conversion of neutrinos inside the neutrino-sphere formed in supernova explosions [38]. Since the magnitude of the Dirac magnetic moments in MESM is about $10^{-20} \mu_B$, it can be concluded that the detection of the magnetic moments of the order of $10^{-14} \mu_B$ would be evidence in favour of that the neutrinos are the Majorana particles. This fact would be of great importance for creating a realistic GUT and for searching the processes, in which there is a violation of the conservation law of total lepton number, for example, for searching the neutrinoless double beta decay.

VI. NEUTRINO MASS OBSERVABLES WITH ALLOWANCE FOR THE STERILE NEUTRINO CONTRIBUTIONS

In order to determine the absolute scale of neutrino masses, it is necessary to determine experimentally at least one of the following values of the neutrino mass observables, namely, the cosmological sum of the neutrino masses m_Σ , the β -decay neutrino mass m_β or the effective (double β -decay) neutrino mass $m_{\beta\beta}$:

$$m_\Sigma = \sum_{i=1}^6 |m_i|, \quad (12a)$$

$$m_\beta = \left(\sum_{i=1}^6 |U_{ei}|^2 m_i^2 \right)^{1/2}, \quad (12b)$$

$$m_{\beta\beta} = \left| \sum_{i=1}^6 U_{ei}^2 m_i \right|. \quad (12c)$$

In general, the neutrino mass observables m_Σ , m_β and $m_{\beta\beta}$, the neutrino masses m_i and $m_{i'}$ and the elements of the neutrino mass matrix M_{ij} can be called as neutrino mass characteristics. To the moment, only the experimental upper limits on the values of the neutrino mass observables are obtained, and they are $m_\Sigma < 0.93 \text{ eV}$ [13, 14] and $m_{\beta\beta} < 0.25 \text{ eV}$ [39]. The results of the experiments in Troitsk and Mainz on the measurement of the electron energy spectrum in the β -decay of tritium give the limitations on the β -decay antineutrino mass m_β ($m_\beta < 2.05 \text{ eV}$ [40], $m_\beta < 2.2 \text{ eV}$ [40, 41]), which is expected to improve to 0.2 eV in the planned KATRIN experiment [41].

Right-handed neutrinos, if they exist, are definitely sterile and are the candidates for the dark matter particles, and at that, it may be several different types of dark matter particles with different masses. In some approximation, the fields ν_R of right-handed neutrinos may be included along with the corresponding fields ν_L in the Dirac mass term in the Lagrangian, so the right-handed neutrinos will be approximately degenerate in mass with ν_L , and then, such neutrinos are light particles with masses less than, at least, 0.3 eV . However, there are still other possibilities for the values of the masses of right-handed neutrinos. One of them is that the masses of ν_R will be much larger than the masses of ν_L , but still of the order of 1 eV . For example, their masses can be between 0.3 eV up to 3 eV . These neutrinos can be called as heavy sterile neutrinos. Neutrinos with masses from 3 eV up to 3 GeV will be called as extra-heavy, and they are super-heavy if their masses are more than 3 GeV . Existence of super-heavy right-handed neutrinos can be used as an explanation for the large quantities of the hidden mass of the Universe, and also to explain the extremely small masses of left-handed neutrinos. Moreover, with the help of super-heavy right-handed neutrinos one can explain the observed baryon asymmetry of the Universe [42]. In this paper we consider the heavy sterile neutrinos

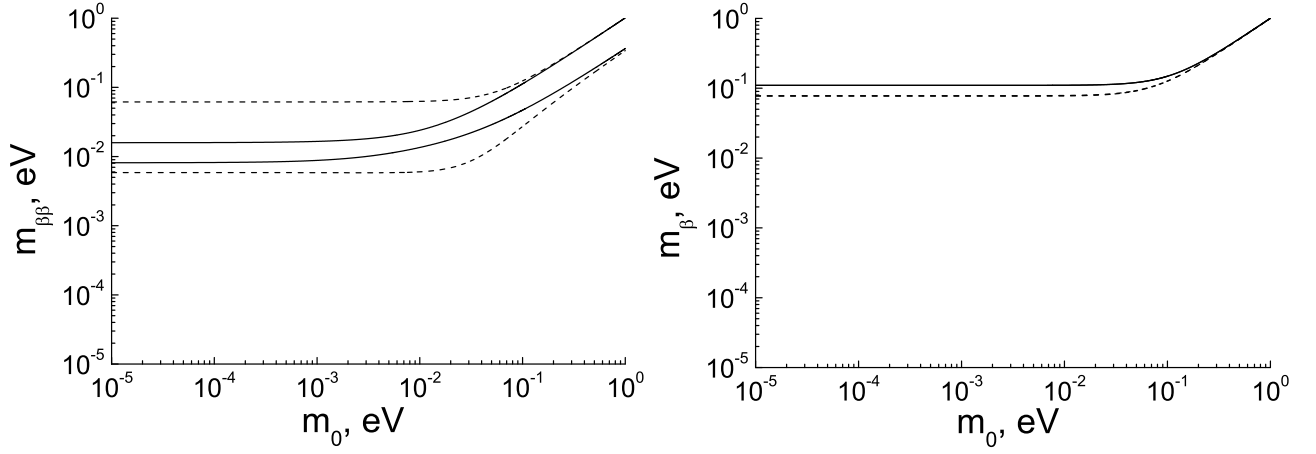


FIG. 1. The ranges of variation of effective neutrino masses $m_{\beta\beta}$ (left panel) and m_{β} (right panel) as a function of the smallest neutrino mass m_0 . The ranges between the solid lines correspond to the NH-case, while the ranges between the dashed lines correspond to the IH-case (in the case of m_{β} , they are extremely narrow and degenerate into lines). $\alpha = 0.1$, $\gamma = 0.1$, $\delta = 0.1$, and $m_* = 0.3$ eV, where $m_* = m_{1'}$ for the NH-case and $m_* = m_{3'}$ for the IH-case, and the values of the masses $m_{1'}$ and $m_{2'}$ are equal in the absolute value to 0.002 eV.

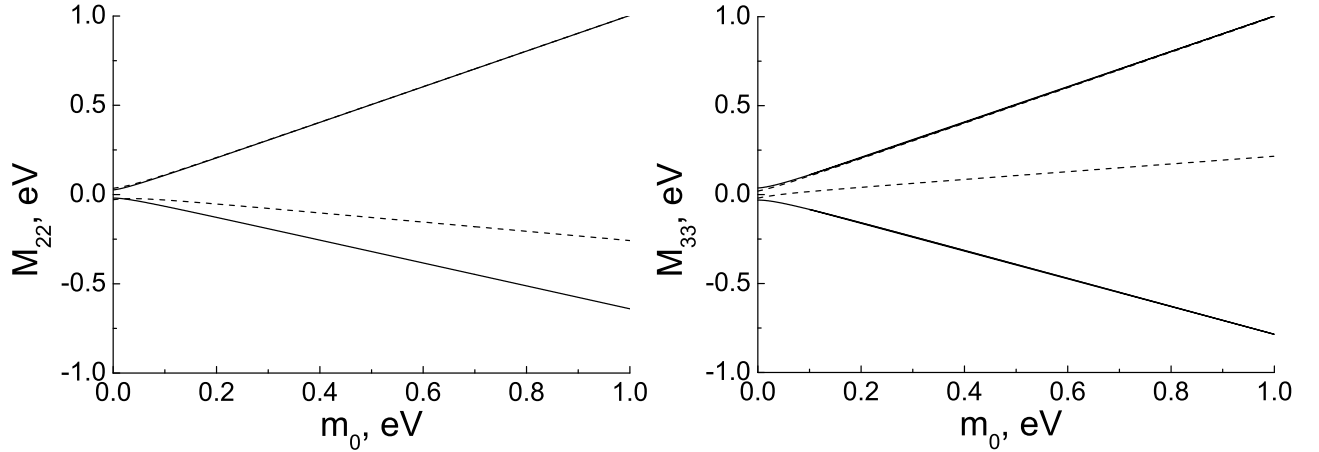


FIG. 2. The ranges of variation of the matrix elements M_{22} and M_{33} of the neutrino mass matrix. The ranges between the solid lines correspond to the NH-case, while the ranges between the dashed lines correspond to the IH-case (with the virtually same lines for *maximal* values in both cases). The parameters are the same as in Fig. 1.

and also light sterile neutrinos, which, together with the active neutrinos, in principle, can form quasi-Dirac neutrinos and which are attractive from a phenomenological point of view [8, 9].

Consider the neutrino mass matrix M_{ij} given by the equation (3). The upper diagonal element of this matrix, i.e., M_{11} , enters a formula for the probability of neutrinoless double beta decay of the nucleus, $(A, Z) \rightarrow (A, Z+2) + 2e$, if the decay occurs with the participation of the light Majorana active neutrino. In this case, the absolute value of M_{11} coincides with $m_{\beta\beta}$ of the equation (12c), i.e., with the effective mass of the neutrino. Half-life time $T_{1/2}^{0\nu 2\beta}$ of the neutrinoless double beta decay is inversely proportional to $m_{\beta\beta}^2$. Note that the detection of the neutrinoless double beta decay is practically the only way to determine whether neutrinos are Dirac or Majorana

particles. The discovery of such a decay would also permit ones to determine the absolute scale of neutrino mass using a measured value of $m_{\beta\beta}$.

We present below the expression for $m_{\beta\beta}$ through the masses of neutrinos and the neutrino mixing parameters for the NH-case and IH-case, respectively:

$$m_{\beta\beta} = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 + s_{13}^2 m_3 + \alpha^2 m_{1'}|, \quad (13a)$$

$$m_{\beta\beta} = |c_{12}^2 c_{13}^2 m_1 + s_{12}^2 c_{13}^2 m_2 + s_{13}^2 m_3 + \gamma^2 m_{3'} + \delta^2 m_{2'} + \delta^2 m_{1'}|. \quad (13b)$$

On the basis of the experimental data given in Eqs. (4a)–(4e) on the oscillation parameters of neutrino mixing and with the help of numerical calculations for the given value of $m_{\beta\beta}$, it is possible to estimate the absolute scale of neu-

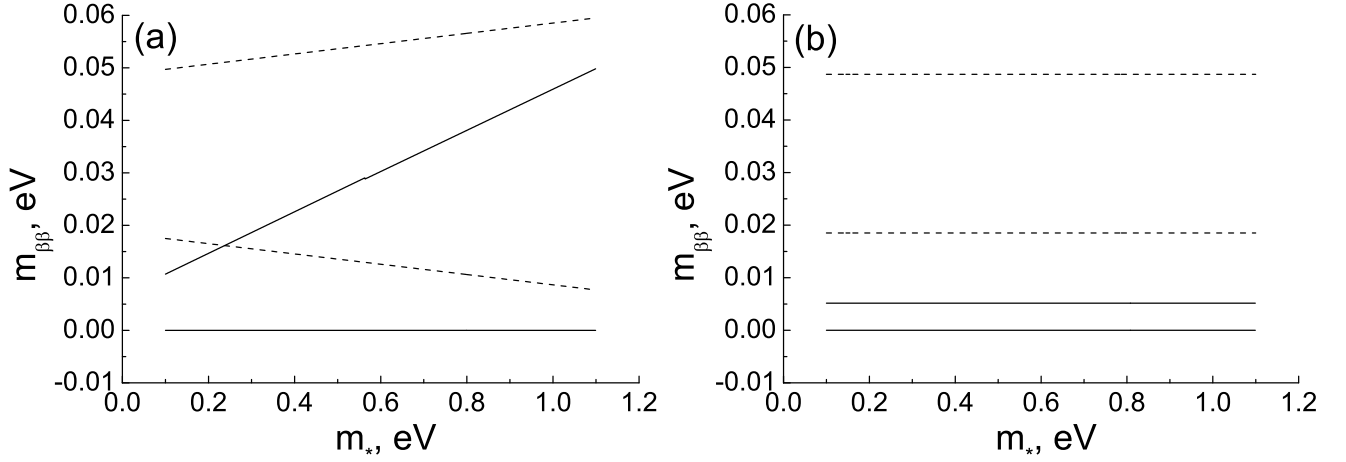


FIG. 3. The ranges of variation of the effective mass of neutrinos $m_{\beta\beta}$, as a function of the greatest mass of sterile neutrinos m_* . The ranges between the solid lines correspond to the NH-case, while the ranges between the dashed lines correspond to the IH-case. On panel (a), the variation of the parameters of sterile neutrinos was performed in the range of $|\alpha| < 0.2$, $|\gamma| < 0.2$ and $|\delta| < 0.2$. For comparison, panel (b) exhibits the case of $\alpha = \gamma = \delta = 0$. For the NH-case, $|m_1| = 0.002$, $|m_2| = 0.0087$, $|m_3| = 0.0497$, $|m_{2'}| = 0.002$ and $|m_{1'}| = 0.002$, while for the IH-case $|m_1| = 0.0496$, $|m_2| = 0.05$, $|m_3| = 0.002$, $|m_{2'}| = 0.002$ and $|m_{1'}| = 0.002$.

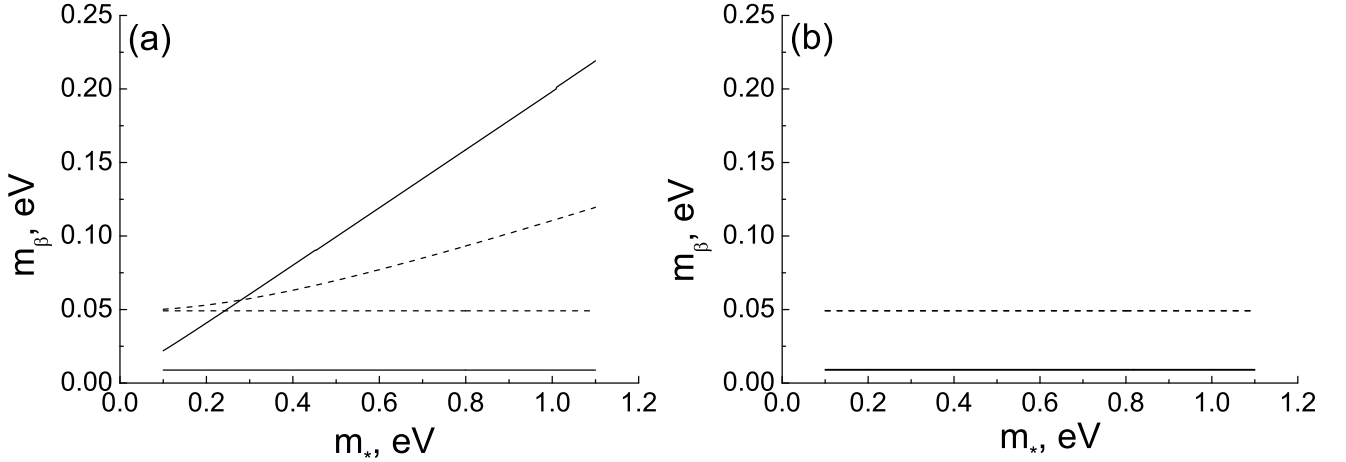


FIG. 4. The ranges of variation of the effective mass of neutrinos m_{β} . The ranges between the solid lines correspond to the NH-case, while the ranges between the dashed lines correspond to the IH-case. On panel (a), the variation of the parameters of sterile neutrinos was performed in the range of $|\alpha| < 0.2$, $|\gamma| < 0.2$ and $|\delta| < 0.2$. For comparison, panel (b) exhibits the case of $\alpha = \gamma = \delta = 0$ (note that here the lines for minimal and maximal values practically coincide in both (NH and IH) cases, so the ranges degenerate into lines). The other parameters are the same as in Fig. 3.

trino mass spectra with normal and inverse hierarchy. Indeed, with using the expression (13) and the experimental data (4) one can determine the explicit dependences of $m_{\beta\beta}$ on the smallest value m_0 from the neutrino masses, i.e., either on m_1 in the NH-case or on m_3 in the IH-case (see Fig. 1, the left panel).

The same can be constructed for the values of other matrix elements M_{ij} , as well as for the values of the neutrino mass observables m_{Σ} and m_{β} (see Figs. 1–2). For example, the right panel of Fig. 1 shows the dependence of m_{β} values on m_0 , while two panels, respectively, of Fig. 2 shows the dependences of M_{22} and M_{33} , on the values of m_0 simultaneously for both NH- and IH-cases.

As was noted above, the matrix elements of the mixing matrix are considered as real in this paper, and, taking into account the results of Ref. [43], it was adopted that $\delta_{CP} \simeq 0$ for the IH-case, while $\delta_{CP} \simeq \pi$ for the NH-case.

It is possible to find and draw the dependences of the neutrino mass observables on the greatest mass m_* among the masses of all sterile neutrinos (see Figs. 3(a)–7(a)). For comparison, Figs. 3(b)–7(b) exhibit the behavior of these features when there are no contributions of the sterile neutrinos. With the help of these graphs, for example, different assumptions about the structure of the mass matrix M and the values of its matrix elements M_{ij} , which were stated in a number of models [44–47]

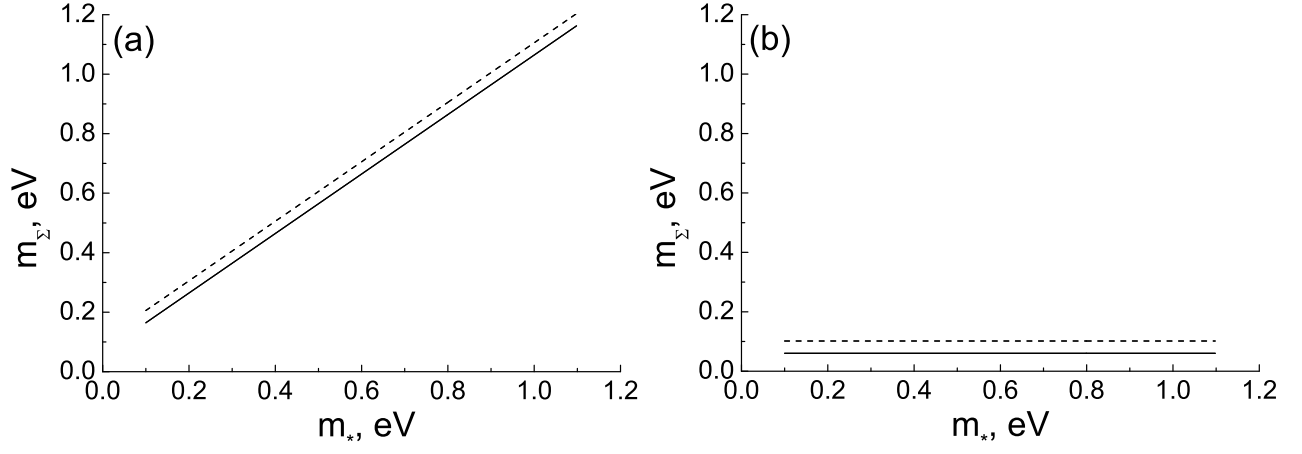


FIG. 5. The ranges of variation of the cosmological effective mass of neutrinos m_Σ (and here the lines for minimal and maximal values practically coincide in both (NH and IH) cases, so the ranges degenerate into lines). The like-lines ranges between the solid lines correspond to the NH-case, while ones between the dashed lines correspond to the IH-case. On panel (a), the variation of the parameters of sterile neutrinos was performed in the range of $|\alpha| < 0.2$, $|\gamma| < 0.2$ and $|\delta| < 0.2$. For comparison, panel (b) exhibits the case of $\alpha = \gamma = \delta = 0$. The other parameters are the same as in Fig. 3.

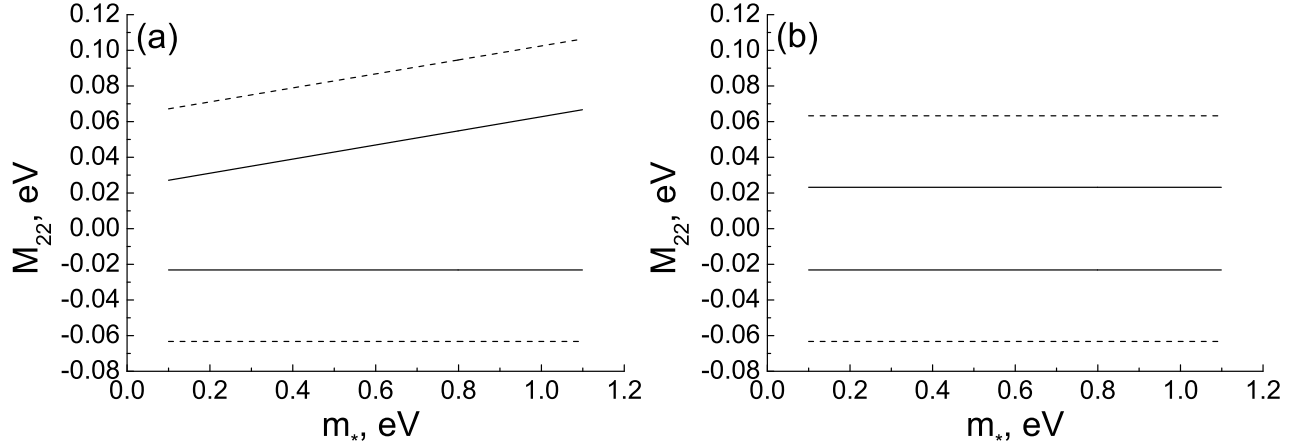


FIG. 6. The ranges of variation of the matrix element M_{22} . The ranges between the solid lines correspond to the NH-case, while ones between the dashed lines correspond to the IH-case. On panel (a), the variation of the parameters of sterile neutrinos was performed in the range of $|\alpha| < 0.2$, $|\gamma| < 0.2$ and $|\delta| < 0.2$. For comparison, panel (b) exhibits the case of $\alpha = \gamma = \delta = 0$. The other parameters are the same as in Fig. 3.

can be tested. Making the necessary calculations with the available experimental data and with minimal arbitrariness, which are reduced to the choice of the values of m_0 , it can be possible to find a range of admissible values of M_{ij} , and thus to check the validity of a model of the structure of the neutrino mass matrix. For example, ranges of values calculated and shown in Fig. 2 for the diagonal matrix elements M_{22} and M_{33} confirm, first of all for the NH-case, the applicability of commonly used approximate equality $M_{22} = M_{33}$, which is a characteristic feature of the μ - τ -symmetry in the neutrino sector [33].

To find the possible experimental effects associated with light sterile neutrinos, it is of interest to consider the changes of the neutrino mass observables due to the possible contributions of sterile neutrinos. For this we

choose as a determinative parameter the greatest mass m_* of the sterile neutrinos, which will be varied in the range from 0.1 eV to 1.1 eV, and then we obtain dependences of the neutrino mass observables m_Σ , m_β and $m_{\beta\beta}$ on m_* taking also into account the contributions of other sterile neutrinos. These dependences are shown in Figs. 3(a)–5(a). Besides, Figs. 6(a)–7(a) shows the dependences of the matrix elements M_{22} and M_{33} on m_* . Although in this paper we consider a sufficiently wide interval of possible values for the mass of the heavy sterile neutrino, there are different estimates of the mass of sterile neutrinos, which result to the values of the order of 1 eV. For example, two corresponding values, namely, 0.46 eV and 0.96 eV were obtained in Ref. [48].

As can be seen from these figures, the effect of sterile neutrinos in the measurements of the mass observables

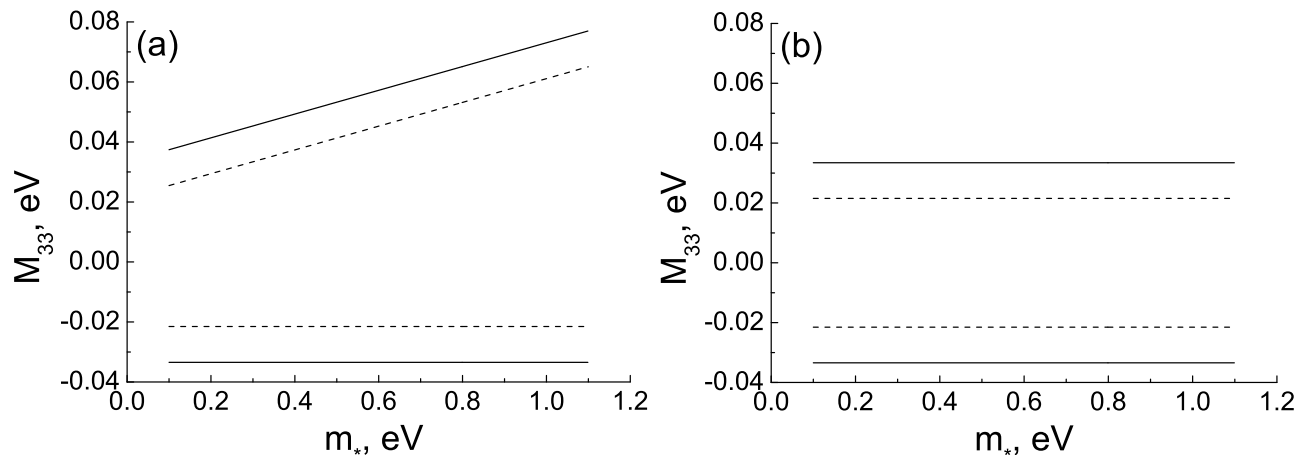


FIG. 7. The ranges of variation of the matrix element M_{33} . The ranges between the solid lines correspond to the NH-case, while ones between the dashed lines correspond to the IH-case. On panel (a), the variation of the parameters of sterile neutrinos was performed in the range of $|\alpha| < 0.2$, $|\gamma| < 0.2$ and $|\delta| < 0.2$. For comparison, panel (b) exhibits the case of $\alpha = \gamma = \delta = 0$. The other parameters are the same as in Fig. 3.

m_Σ , m_β and $m_{\beta\beta}$ can not be detected at the sterile neutrinos masses less than 0.5 eV, if the condition of exceeding the background level by more than 2σ is supposed to be fulfilled. On the other hand, the recent results of measurements of the cosmological microwave background lead to the fact that the sum of the masses of all types of neutrinos does not exceed 0.5 eV [14]. Thus, the search of the effect of sterile neutrinos at the $(3 \div 5)\sigma$ level should be performed in the oscillation experiments, but not in the experiments for determination of the absolute scale of neutrino masses.

The obtained ranges of possible values of m_Σ , m_β and $m_{\beta\beta}$ can be used in both the planning and the interpretation of the results of the experiments to search for the neutrinoless double beta decay, in the determination of the β -decay neutrino mass, as well as in the determination of the cosmological sum of the absolute values of masses of all neutrinos.

VII. CONCLUSION

Properties of neutrinos are very mysterious and intensive theoretical and experimental studies to determine the nature and characteristics of these elementary particles are required. Construction and development of adequate phenomenological models of neutrinos, which can generalize the SM in the neutrino sector, is one of the ways for correct interpretation and prediction of the experimental results and successful searching the GUT. In the current, the priority of the experimental and theoretical researches in the neutrino physics is to verify the existence of light and heavy sterile neutrinos, to determine their number, as well as to determine the absolute

mass scale, both for active and sterile neutrinos [49].

In the current work, the phenomenological $(3 + 1 + 2)$ -model of neutrino with three active and three sterile neutrinos, one of which is more massive and the other two sterile neutrinos are much lighter is used to study the properties of active and sterile neutrinos. The model allows reducing the number of sterile neutrinos in case if the model-independent experimental restrictions on their number will be established. On the basis of recent experimental data, the allowable ranges of the neutrino mass observables m_Σ , m_β and $m_{\beta\beta}$ have been calculated taking into account the contributions of the sterile neutrinos. Using the estimates of the masses of active neutrinos, the dependences of mass observables m_Σ , m_β and $m_{\beta\beta}$ on the mass of one of sterile neutrino, which was varied in the range of possible values from 0.1 eV to 1.1 eV, were depicted. Currently, for experimental determination of the neutrino mass observables m_Σ , m_β and $m_{\beta\beta}$ the numerous experiments are both carried out and planned on searching for the neutrinoless double beta decay, defining the form of beta-spectrum in the decay of tritium, as well as the cosmological observations. To interpret and to predict the results of these experiments, and also to explain the LSND/MiniBooNE, reactor and gallium anomalies, the considered model of neutrinos and the obtained values of the neutrino mass observables can be used.

ACKNOWLEDGMENTS

The authors are grateful to Yu. S. Lyutostansky, V. P. Martemyanov, M. D. Skorokhvatov, S. V. Sukhotin, S. V. Semenov, D. K. Nadezhin and I. V. Panov for useful discussions. This work was partially supported by grants RFBR 11-02-00882-a and 12-02-12116-ofi-m.

REFERENCES

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- [1] J. Beringer *et al.* (Particle Data Group), Phys. Rev. D **86**, 010001 (2012).
- [2] G. L. Fogli *et al.*, Phys. Rev. D **86**, 013012 (2012); arXiv:1205.5254v3 [hep-ph].
- [3] T. Schwetz, M. Tórtola, and J. W. F. Valle, New J. Phys. **13** 063004 (2011); arXiv:1103.0734v2 [hep-ph].
- [4] G. C. Branco, R. González Felipe, and F. R. Joaquim, Rev. Mod. Phys. **84**, 515 (2012).
- [5] M. C. Gonzalez-Garcia and Y. Nir, Rev. Mod. Phys. **75**, 345 (2003).
- [6] C. Giunti, M. Laveder, Y. F. Li, Q. Y. Liu, and H. W. Long, Phys. Rev. D **86**, 113014 (2012).
- [7] J. Kopp, P. A. N. Machado, M. Maltoni, and T. Schwetz, J. High Energy Phys. 05 (2013) 050.
- [8] N. Yu. Zysina, S. V. Fomichev, and V. V. Khrushov, arXiv:1209.0545 [hep-ph].
- [9] N. Yu. Zysina, S. V. Fomichev, and V. V. Khrushov, Perspektivnye materialy, special issue 14, 435 (2013).
- [10] M. Duerr, P. F. Pérez, and M. Lindner, Phys. Rev. D **88**, 051170(R) (2013); arXiv:1306.0568 [hep-ph].
- [11] J. M. Conrad, C. M. Ignarra, G. Karagiorgi, M. H. Shaevitz, and J. Spitz, Adv. High Energy Phys., 163897 (2013).
- [12] P. C. de Holanda and A. Yu. Smirnov, Phys. Rev. D **83**, 113011 (2011).
- [13] E. Komatsu *et al.* [WMAP Collab.], Astrophys. J. Suppl. **192**, 18 (2011).
- [14] P. A. R. Ade *et al.* [Planck Collab.], arXiv:1303.5076 [astro-ph.CO].
- [15] D. Boyanovsky and C.-M. Ho, J. High Energy Phys. 07 (2007) 030.
- [16] S. Hannestad, I. Tamborra, and T. Tram, J. Cosmol. Astropart. Phys. 07 (2012) 025.
- [17] A. Mirizzi, N. Saviano, G. Miele, and P. D. Serpico, Phys. Rev. D **86**, 053009 (2012).
- [18] K. N. Abazajian *et al.*, arXiv:1204.5379 [hep-ph].
- [19] J. M. Conrad, W. C. Louis, and M. H. Shaevitz, Annu. Rev. Nucl. Part. Sci. **63** 45 (2013); arXiv:1306.6494 [hep-ex].
- [20] S. M. Bilenky, C. Giunti, and W. Grimus, Prog. Part. Nucl. Phys. **43**, 1 (1999).
- [21] A. Aguilar *et al.* [LSND Collab.], Phys. Rev. D **64**, 112007 (2001).
- [22] A. A. Aguilar-Arevalo *et al.* [MiniBooNE Collab.], Phys. Rev. Lett. **105**, 181801 (2010).
- [23] Th. A. Mueller *et al.*, Phys. Rev. C **83**, 054615 (2011).
- [24] G. Mention *et al.*, Phys. Rev. D **83**, 073006 (2011).
- [25] J. N. Abdurashitov *et al.* [SAGE Collab.], Phys. Rev. C **80**, 015807 (2009).
- [26] F. Kaether *et al.*, Phys. Lett. B **685**, 47 (2010).
- [27] C. Giunti and M. Laveder, Phys. Rev. C **83**, 065504 (2011).
- [28] V. V. Khrushov and S. V. Fomichev, arXiv:1310.5817 [hep-ph].
- [29] A. Palazzo, Phys. Rev. D **85** 077301 (2012); arXiv:1201.4280 [hep-ph].
- [30] R. N. Mohapatra and A. Y. Smirnov, Annu. Rev. Nucl. Part. Sci. **56**, 569 (2006); arXiv:hep-ph/0603118.
- [31] H. Fritzsch, arXiv:0902.2817 [hep-ph].
- [32] Yu. V. Gaponov, Yad. Fiz. **74**, 290 (2011) [Phys. Atom. Nucl. **74**, 272 (2011)].
- [33] V. V. Khrushov, Yad. Fiz. **76**, 1421 (2013) [Phys. Atom. Nucl. **76**, 1356 (2013)].
- [34] A. I. Aleshin *et al.*, Preprint IAE-6755/2, Moscow (2013).
- [35] K. Fujikawa, B. W. Lee, and A. I. Sanda, Phys. Rev. D **6**, 2923 (1972).
- [36] K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. **45**, 963 (1980).
- [37] M. Czakon, J. Gluza, and M. Zralek, Phys. Rev. D **59**, 013010 (1999).
- [38] A. de Gouvêa and S. Shalgar, J. Cosmol. Astropart. Phys. 04 (2013) 018; arXiv:1301.5637 [astro-ph.HE].
- [39] A. Gando *et al.* [KamLAND-Zen Collab.], Phys. Rev. Lett. **110**, 062502 (2013).
- [40] V. M. Lobashev, Nucl. Phys. A **719**, 153c (2003).
- [41] E. W. Otten and C. Weinheimer, Rept. Prog. Phys. **71**, 086201 (2008).
- [42] M. Fukugita and T. Yanagida, Phys. Lett., B **174**, 45 (1986).
- [43] D. V. Forero, M. Tórtola, and J. W. F. Valle, Phys. Rev. D **86** 073012 (2012); arXiv:1205.4018v4 [hep-ph].
- [44] H. Fritzsch, Z.-z. Xing, and S. Zhou, J. High Energy Phys. 09 (2011) 083; arXiv:1108.4534 [hep-ph].
- [45] G. Blankenburg and D. Meloni, Nucl. Phys. B **867**, 749 (2013); arXiv:1204.2706 [hep-ph].
- [46] B. C. Chauhan, J. Pulido, and M. Picariello, Phys. Rev. D **73**, 053003 (2006).
- [47] A. Merle and W. Rodejohann, Phys. Rev. D **73**, 073012 (2006).
- [48] V. V. Sinev, arXiv:1103.2452v3 [hep-ex].
- [49] G. J. Barker *et al.*, arXiv:1309.7810 [hep-ex].